Econ 802

Second Midterm Exam

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All questions have equal weight. If something is unclear, please ask. You may want to work first on the questions where you feel most confident.

1. A competitive firm has the cost function c(w, y) where $w = (w_1 \,.\,.\, w_n) > 0$ is the input price vector and $y > 0$ is output (a scalar).

(a) Consider (w, x) and (w′, x′) where x is a cost-minimizing way to produce y at prices w and x′ is a cost-minimizing way to produce y (the same output) at prices w′. Prove that $c(tw + (1\text{-}t)w', y) \geq tc(w, y) + (1\text{-}t)c(w', y)$ for $0 \leq t \leq 1$ and briefly interpret this result.

(b) Prove that the conditional input demand $x_i(w,y)$ for input i cannot be an increasing function of its own price w_i.

(c) Hold input prices w fixed and consider c(w, y) as a function of output (y). Let the output price be p. Find the first and second order necessary conditions for profit maximization with respect to y (assume an interior solution). On a graph, show a case where the first order condition holds but the second order condition does not, and explain briefly.

2. Kaitlin has the utility function u = g(x) where $x = (x_1 \,.\,.\, x_J) \geq 0$ is her consumption bundle. The function g is homogeneous of degree one and strictly quasi-concave. You can ignore Kuhn-Tucker multipliers.

(a) Assume the price vector $p > 0$ is held constant. Prove that if x^0 is optimal at the income m^0, then tx^0 is optimal at the income tm^0 (where $t > 0$ is a scalar).

(b) Assume the result in (a) is true. Prove that the indirect utility function has the form v(p, m) = v(p, 1)m.

(c) Assume the results in (a) and (b) are true. Supose there are many consumers i = 1 . . n who do not have identical preferences, but they all have utility functions that are homogeneous of degree one. Does this imply that the aggregate Marshallian demands depend only on aggregate income? Explain your reasoning carefully.

3. For each of the following utility functions (i) derive the Marshallian demand for good one; (ii) compute the partial derivative $\partial x_1/\partial p_1$; and (iii) use the Slutsky equation to break this partial derivative into a substitution effect and an income effect. DO NOT compute Hicksian (or compensated) demand functions.

(a) $u = \min\{ax_1, bx_2\}$ $\quad a > 0, b > 0$

(b) $u = a \ln x_1 + b \ln x_2$ $\quad a > 0, b > 0$

(c) $u = x_1^a + x_2$ $\quad 0 < a < 1$

4. When the price vector was $p^0 > 0$, George consumed the bundle $x^0 \geq 0$.

(a) Due to a change in government regulations, the new price vector is $p^* > 0$. George can change to a new consumption bundle x^* if he wants to. How much money income m^* does George need to be exactly as well off as he was in the original situation? Explain carefully using both math and a graph.

(b) Suppose we don't know what bundle x^0 George originally had, but we know that his original income level was m^0 when the prices were p^0. Does this change your answer to the question in (a)? Explain carefully using both math and a graph.

(c) Make the same assumptions as in part (b). Is it possible to have $m^* > m^0$? What about $m^* = m^0$? What about $m^* < m^0$? Explain carefully using math and a graph.

5. Assume Esmeralda chooses the bundle of goods $x^* > 0$ when the price vector is $p^* > 0$ and her income is $m^* = 1$.

(a) For the case where there are only two goods, suppose Esmeralda switches to the bundle $x' \neq x^*$ after the prices have changed to $p' \neq p^*$, where $p'x' = p'x^* = m^* = 1$. Does this violate the generalized axiom of revealed preference (GARP)? Why or why not?

(b) For the case where there are only two goods (x_1, x_2), prove that the indirect utility function satisfies $v(p^*,1) \leq v(p,1)$ for all price vectors such that $px^* = 1$. Explain your argument using a graph.

(c) Suppose there are n goods and you know the utility function $u(x)$, which is strictly quasi-concave. Explain how you would solve mathematically for the price vector p^* as a function of the goods vector x^* (you can continue to assume that $m^* = 1$, and you can assume that all solutions are interior).